Lecture 25

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ORGANIZATION

I. Voltage Source Inverter (VSI)

A. Six-Step VSI

B. Pulse-Width Modulated VSI

II. PWM Methods

A. Sine PWM

B. Hysteresis (Bang-bang)

C. Space Vector PWM

III. References

I. Voltage Source Inverter (VSI)
A. Six-Step VSI (1)

- **Six-Step three-phase Voltage Source Inverter**

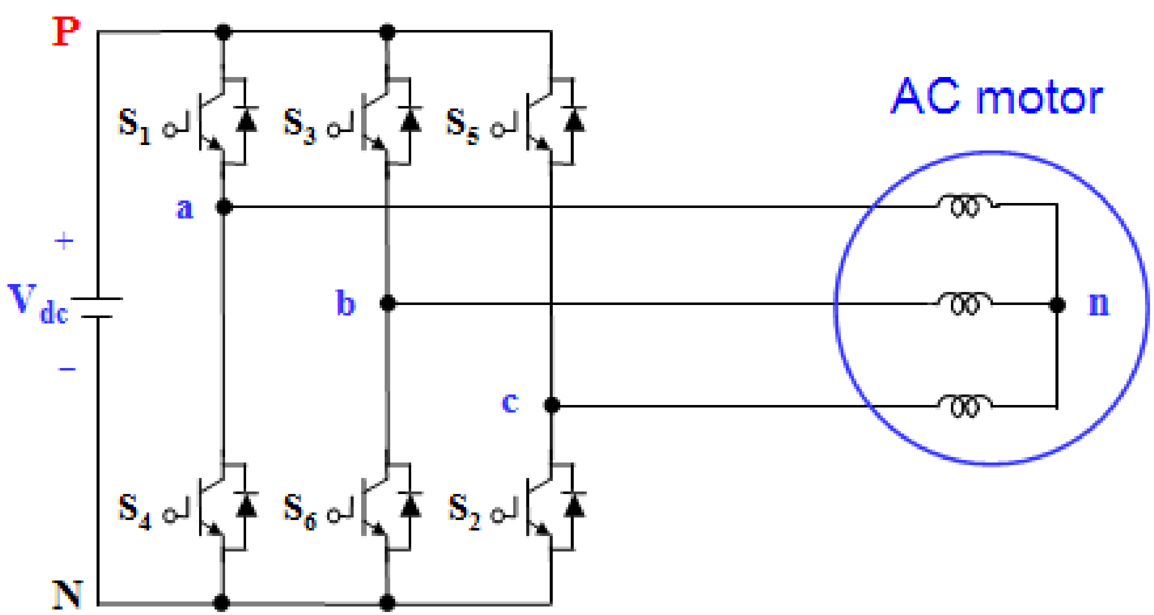


Fig. 1 Three-phase voltage source inverter.

I. Voltage Source Inverter (VSI)
A. Six-Step VSI (2)

- Gating signals, switching sequence and line to negative voltages

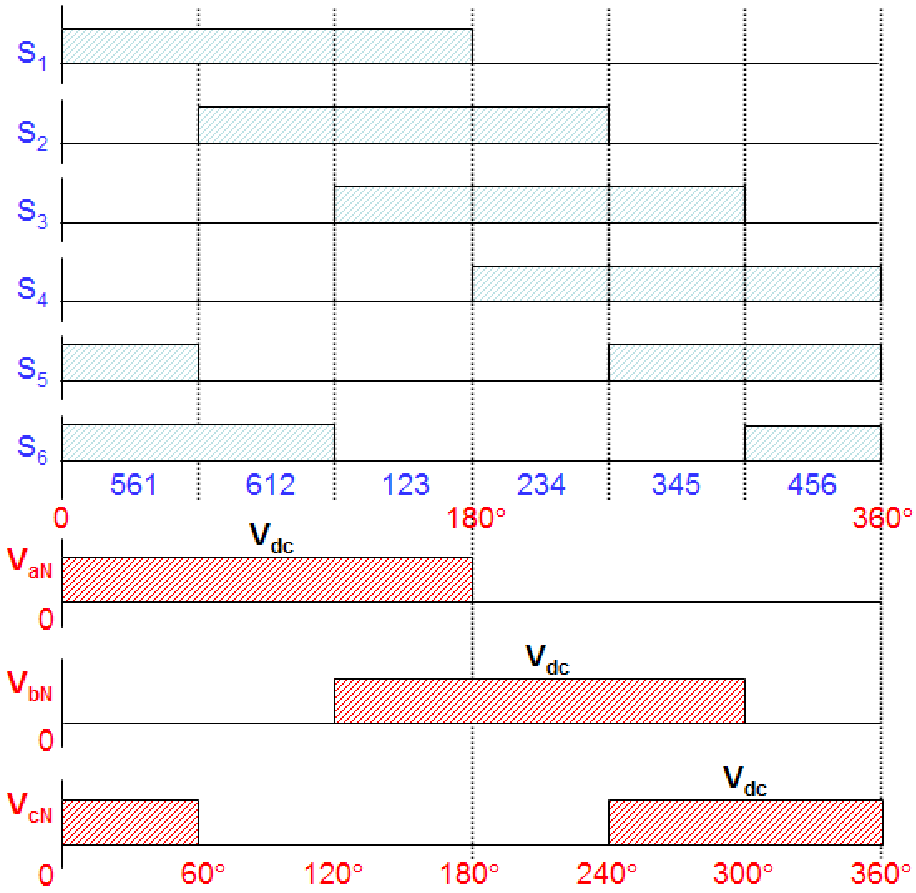


Fig. 2 Waveforms of gating signals, switching sequence, line to negative voltages for six-step voltage source inverter.

I. Voltage Source Inverter (VSI)
A. Six-Step VSI (3)

- **Switching Sequence:**

  561 (V_1) → 612 (V_2) → 123 (V_3) → 234 (V_4) → 345 (V_5) → 456 (V_6) → 561 (V_1)

  where, 561 means that S_5, S_6 and S_1 are switched on

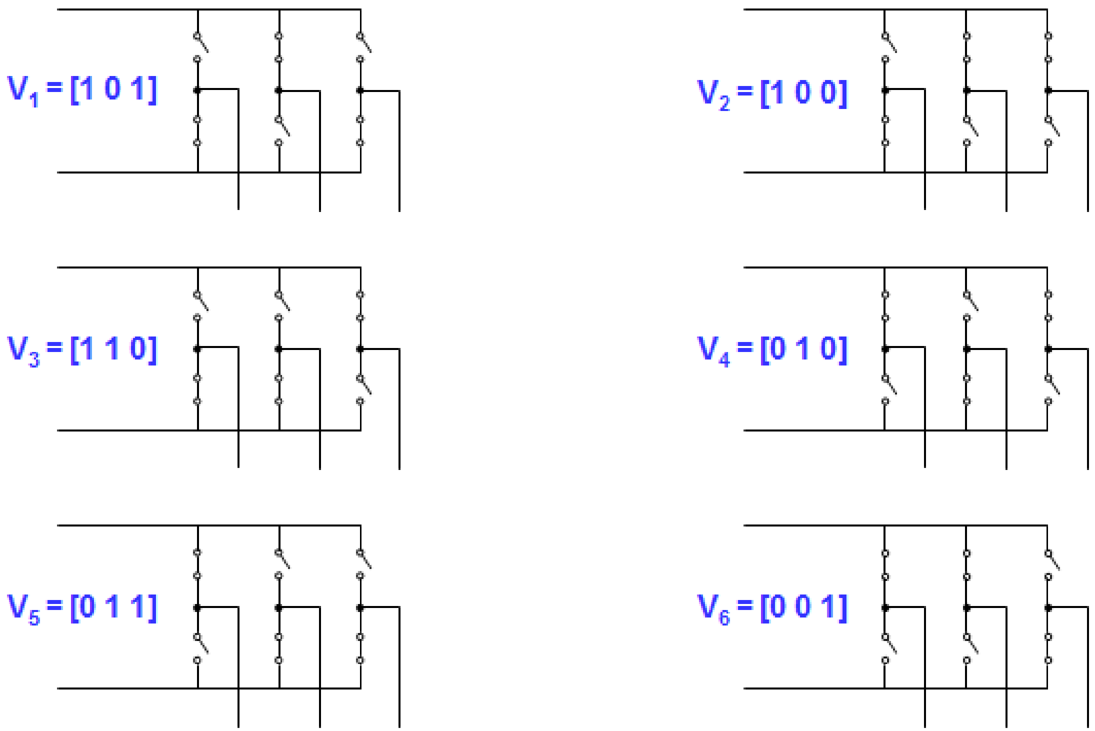


Fig. 3 Six inverter voltage vectors for six-step voltage source inverter.

I. Voltage Source Inverter (VSI)

A. Six-Step VSI (4)

➢ **Line to line voltages (V_{ab}, V_{bc}, V_{ca}) and line to neutral voltages (V_{an}, V_{bn}, V_{cn})**

- **Line to line voltages**
  - ⇨ $V_{ab} = V_{aN} - V_{bN}$
  - ⇨ $V_{bc} = V_{bN} - V_{cN}$
  - ⇨ $V_{ca} = V_{cN} - V_{aN}$
- **Phase voltages**
  - ⇨ $V_{an} = 2/3V_{aN} - 1/3V_{bN} - 1/3V_{cN}$
  - ⇨ $V_{bn} = -1/3V_{aN} + 2/3V_{bN} - 1/3V_{cN}$
  - ⇨ $V_{cn} = -1/3V_{aN} - 1/3V_{bN} + 2/3V_{cN}$

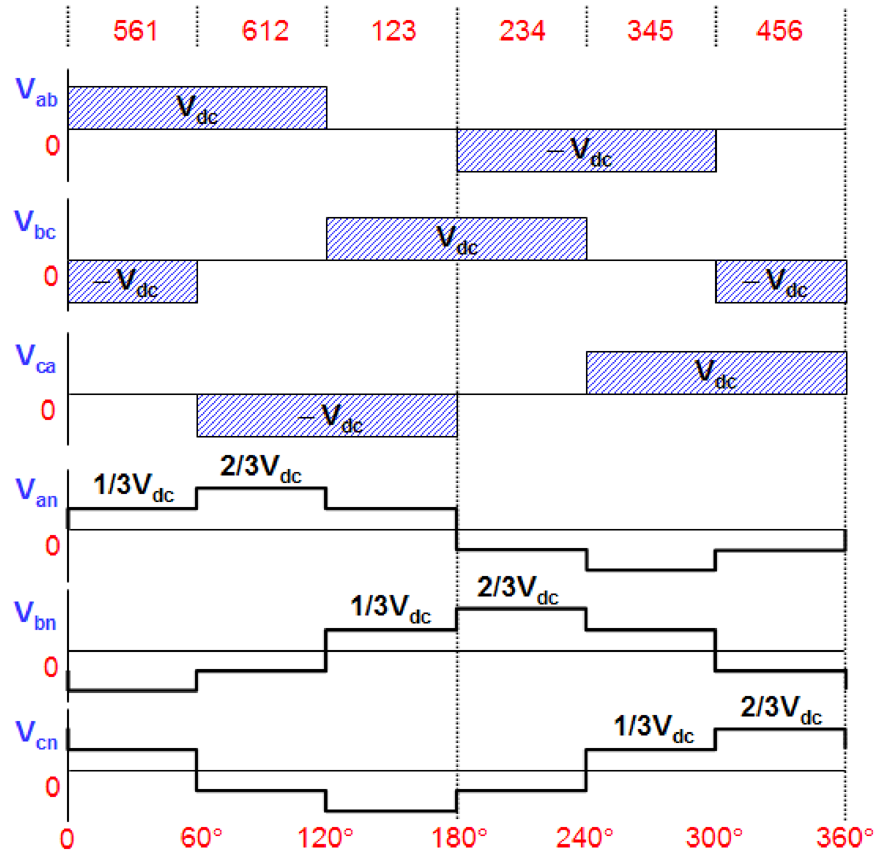


Fig. 4 Waveforms of line to neutral (phase) voltages and line to line voltages for six-step voltage source inverter.

I. Voltage Source Inverter (VSI)
A. Six-Step VSI (5)

- **Amplitude of line to line voltages (V_{ab}, V_{bc}, V_{ca})**
  - **Fundamental Frequency Component $(V_{ab})_1$**

$$(\mathbf{V_{ab}})_1(\mathbf{rms}) = \frac{\sqrt{3}}{\sqrt{2}}\frac{4}{\pi}\frac{V_{dc}}{2} = \frac{\sqrt{6}}{\pi}V_{dc} \approx 0.78V_{dc}$$

  - **Harmonic Frequency Components $(V_{ab})_h$**

: amplitudes of harmonics decrease inversely proportional to their harmonic order

$$(\mathbf{V}_{ab})_{\mathbf{h}}(\mathbf{rms}) = \frac{0.78}{h}V_{dc}$$

where, $\quad h = 6n \pm 1 \quad (n = 1, 2, 3, \ldots..)$

I. Voltage Source Inverter (VSI)
A. Six-Step VSI (6)

- **Characteristics of Six-step VSI**

  - **It is called "six-step inverter" because of the presence of six "steps" in the line to neutral (phase) voltage waveform**

  - **Harmonics of order three and multiples of three are absent from both the line to line and the line to neutral voltages and consequently absent from the currents**

  - **Output amplitude in a three-phase inverter can be controlled by only change of DC-link voltage (V_{dc})**

I. Voltage Source Inverter (VSI)
B. Pulse-Width Modulated VSI (1)

- **Objective of PWM**
  - **Control of inverter output voltage**
  - **Reduction of harmonics**

- **Disadvantages of PWM**
  - **Increase of switching losses due to high PWM frequency**
  - **Reduction of available voltage**
  - **EMI problems due to high-order harmonics**

I. Voltage Source Inverter (VSI)
B. Pulse-Width Modulated VSI (2)

➢ **Pulse-Width Modulation (PWM)**

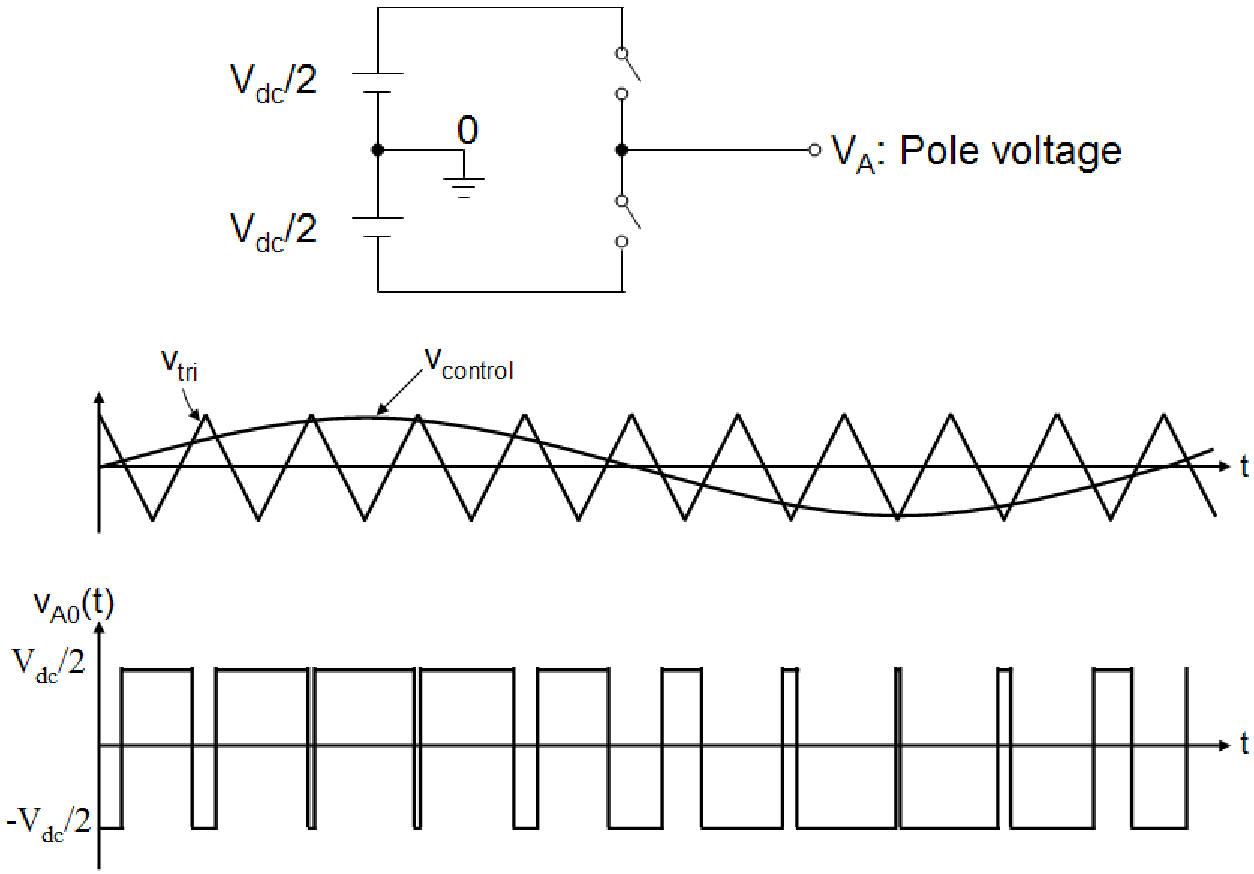


Fig. 5 Pulse-width modulation.

I. Voltage Source Inverter (VSI)
B. Pulse-Width Modulated VSI (3)

- **Inverter output voltage**
  - **When $v_{control} > v_{tri}$, $V_{A0} = V_{dc}/2$**
  - **When $v_{control} < v_{tri}$, $V_{A0} = -V_{dc}/2$**
- **Control of inverter output voltage**
  - **PWM frequency is the same as the frequency of v_{tri}**
  - **Amplitude is controlled by the peak value of $v_{control}$**
  - **Fundamental frequency is controlled by the frequency of $v_{control}$**
- **Modulation Index (m)**

$$\therefore m = \frac{v_{control}}{v_{tri}} = \frac{peak \quad of \quad (V_{A0})_1}{V_{dc}/2},$$

where, $(V_{A0})_1$: fundamenta l frequecny component of V_{A0}

II. PWM METHODS
A. Sine PWM (1)

➢ **Three-phase inverter**

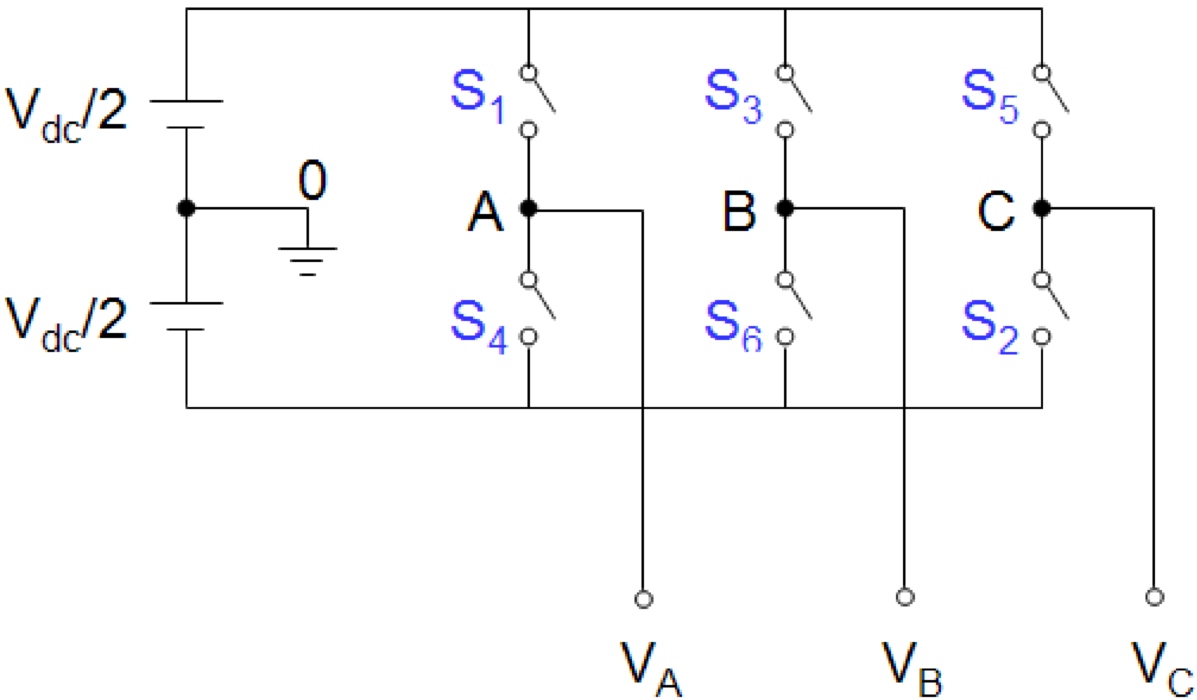


Fig. 6 Three-phase Sine PWM inverter.

II. PWM METHODS
A. Sine PWM (2)

➢ **Three-phase sine PWM waveforms**

- **Frequency of v_{tri} and $v_{control}$**

  ⇨ **Frequency of v_{tri} = f_s**

  ⇨ **Frequency of $v_{control}$ = f_1**

  where, f_s = PWM frequency

  f_1 = Fundamental frequency

- **Inverter output voltage**

  ⇨ **When $v_{control} > v_{tri}$, $V_{A0} = V_{dc}/2$**

  ⇨ **When $v_{control} < v_{tri}$, $V_{A0} = -V_{dc}/2$**

  where, $V_{AB} = V_{A0} - V_{B0}$

  $V_{BC} = V_{B0} - V_{C0}$

  $V_{CA} = V_{C0} - V_{A0}$

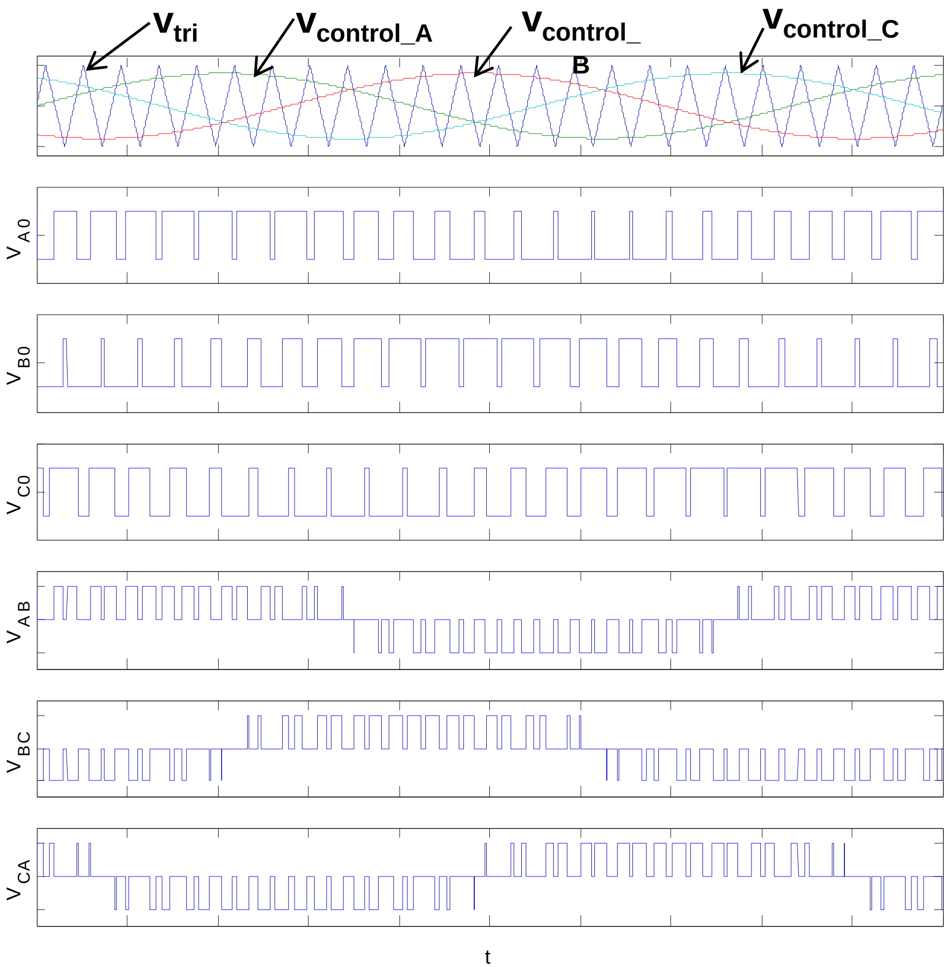


Fig. 7 Waveforms of three-phase sine PWM inverter.

II. PWM METHODS
A. Sine PWM (3)

➢ **Amplitude modulation ratio (m_a)**

$$\therefore m_a = \frac{peak \quad amplitude \quad of \quad v_{control}}{amplitude \quad of \quad v_{tri}} = \frac{peak \quad value \quad of \quad (V_{A0})_1}{V_{dc}/2},$$

where, $(\mathrm{V_{A0}})_1$: fundamental frequecny component of $\mathrm{V_{A0}}$

➢ **Frequency modulation ratio (m_f)**

$$m_f = \frac{f_s}{f_1}, \quad \text{where,} \quad \mathrm{f_s} = \text{PWM frequency and } \mathrm{f_1} = \text{fundamental frequency}$$

- **m_f should be an odd integer**
  - ⇨ **if m_f is not an integer, there may exist sunhamonics at output voltage**
  - ⇨ **if m_f is not odd, DC component may exist and even harmonics are present at output voltage**

- **m_f should be a multiple of 3 for three-phase PWM inverter**
  - ⇨ **An odd multiple of 3 and even harmonics are suppressed**

II. PWM METHODS
B. Hysteresis (Bang-bang) PWM (1)

➢ **Three-phase inverter for hysteresis Current Control**

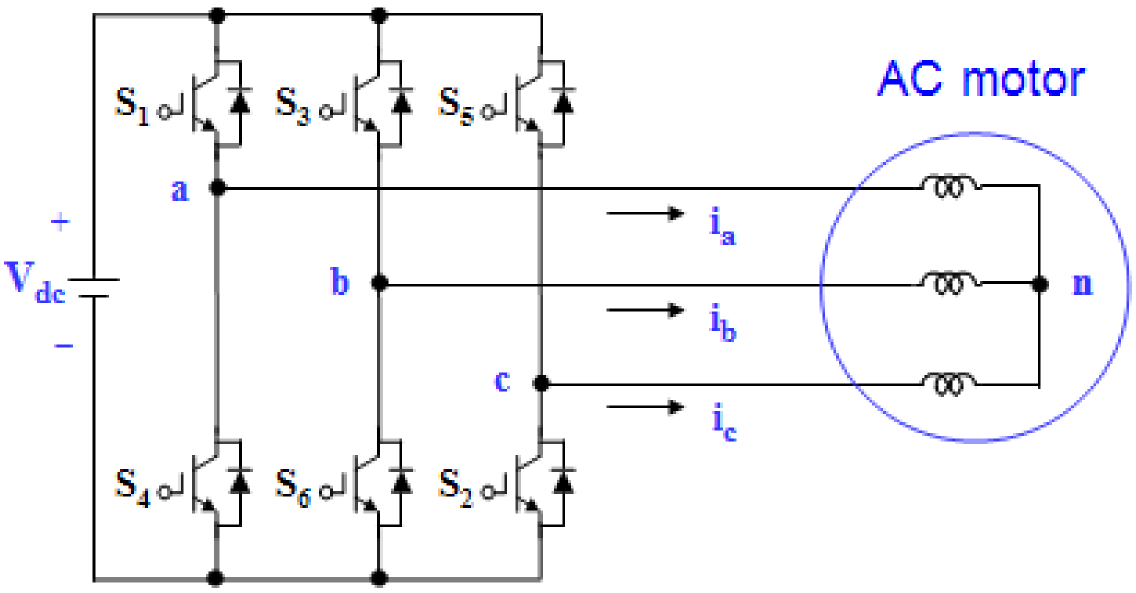


Fig. 8 Three-phase inverter for hysteresis current control.

II. PWM METHODS
B. Hysteresis (Bang-bang) PWM (2)

- **Hysteresis Current Controller**

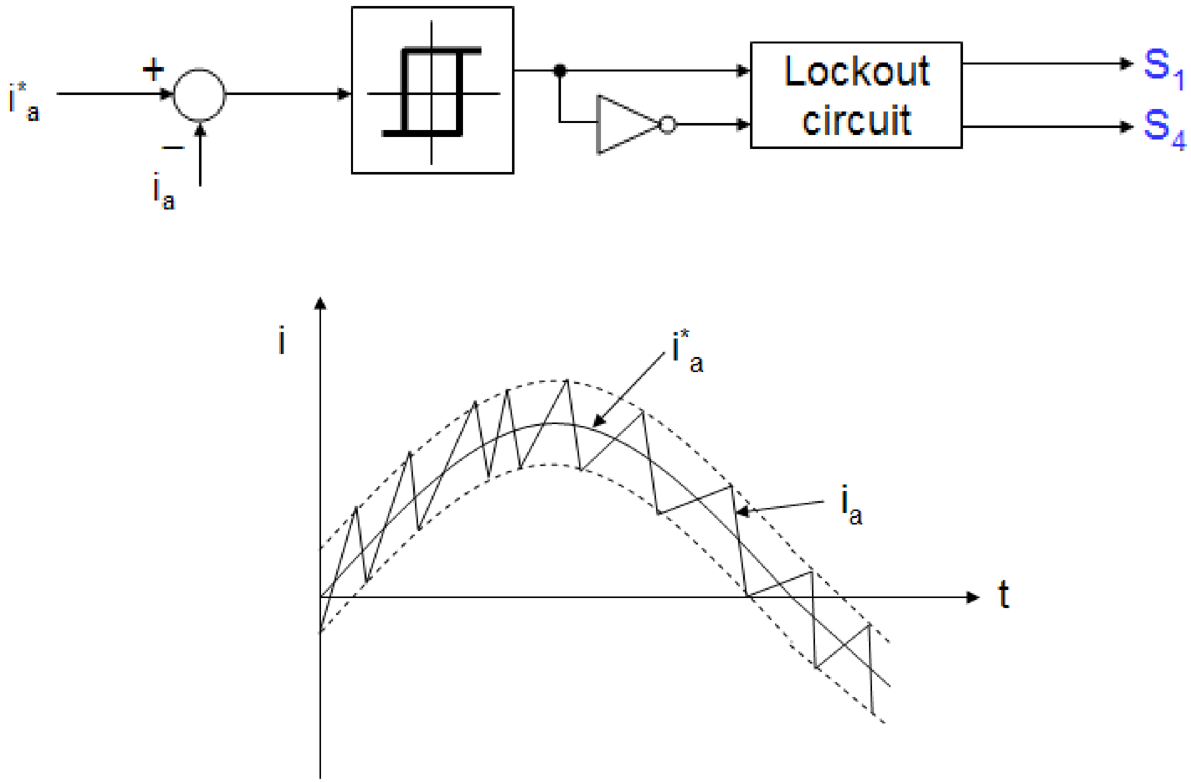


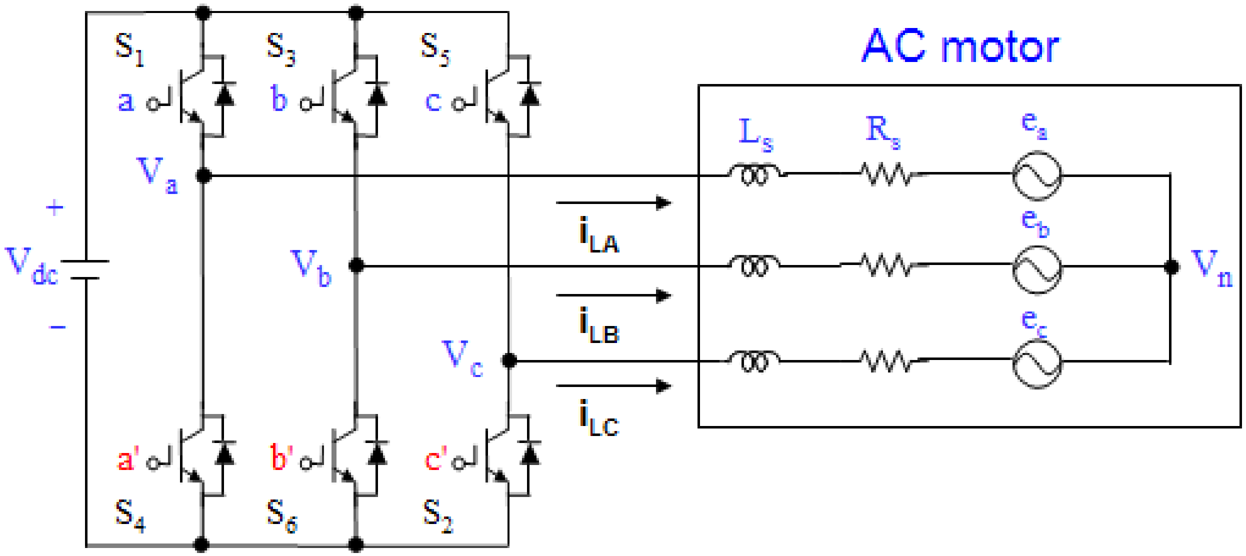
Fig. 9 Hysteresis current controller at Phase “a”.

II. PWM METHODS
B. Hysteresis (Bang-bang) PWM (3)

- **Characteristics of hysteresis Current Control**
  - **Advantages**
    - ⇨ **Excellent dynamic response**
    - ⇨ **Low cost and easy implementation**
  - **Drawbacks**
    - ⇨ **Large current ripple in steady-state**
    - ⇨ **Variation of switching frequency**
    - ⇨ **No intercommunication between each hysterisis controller of three phases and hence no strategy to generate zero-voltage vectors. As a result, the switching frequency increases at lower modulation index and the signal will leave the hysteresis band whenever the zero vector is turned on.**
    - ⇨ **The modulation process generates subharmonic components**

II. PWM METHODS
C. Space Vector PWM (1)

➢ **Output voltages of three-phase inverter (1)**



where, upper transistors: S_1, S_3, S_5
lower transistors: S_4, S_6, S_2
switching variable vector: a, b, c

Fig. 10 Three-phase power inverter.

II. PWM METHODS
C. Space Vector PWM (2)

➢ **Output voltages of three-phase inverter (2)**

- **S_1 through S_6 are the six power transistors that shape the ouput voltage**
- **When an upper switch is turned on (i.e., a, b or c is "1"), the corresponding lower switch is turned off (i.e., a', b' or c' is "0")**

  ⇨ **Eight possible combinations of on and off patterns for the three upper transistors (S_1, S_3, S_5)**

- **Line to line voltage vector $[V_{ab}\ V_{bc}\ V_{ca}]^t$**

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = V_{dc} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \quad \text{where switching variable vector } [a \quad b \quad c]^t$$

- **Line to neutral (phase) voltage vector $[V_{an}\ V_{bn}\ V_{cn}]^t$**

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \frac{1}{3} V_{dc} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

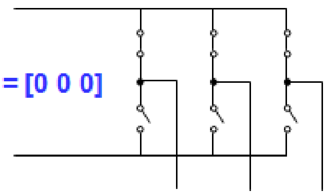
II. PWM METHODS

C. Space Vector PWM (3)

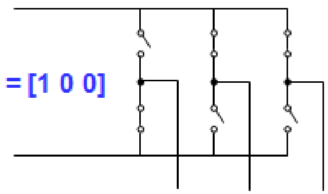
- **Output voltages of three-phase inverter (3)**

  - **The eight inverter voltage vectors (V_0 to V_7)**

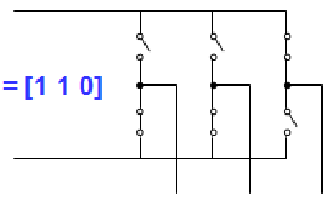
$V_0 = [0\ 0\ 0]$



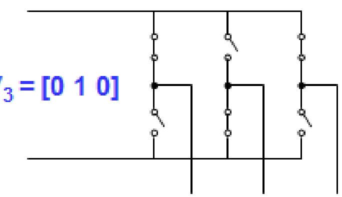
$V_1 = [1\ 0\ 0]$



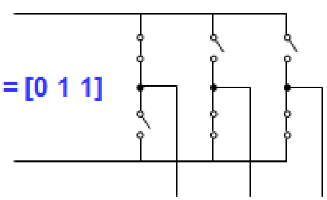
$V_2 = [1\ 1\ 0]$



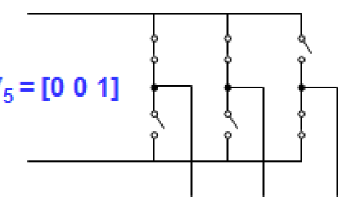
$V_3 = [0\ 1\ 0]$



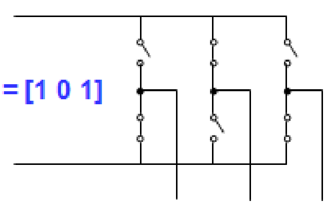
$V_4 = [0\ 1\ 1]$



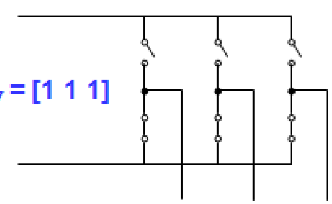
$V_5 = [0\ 0\ 1]$



$V_6 = [1\ 0\ 1]$



$V_7 = [1\ 1\ 1]$



II. PWM METHODS

C. Space Vector PWM (4)

- **Output voltages of three-phase inverter (4)**
  - **The eight combinations, phase voltages and output line to line voltages**

Voltage Vectors	Switching Vectors			Line to neutral voltage			Line to line voltage		
	a	b	c	V_{an}	V_{bn}	V_{cn}	V_{ab}	V_{bc}	V_{ca}
V_0	0	0	0	0	0	0	0	0	0
V_1	1	0	0	2/3	-1/3	-1/3	1	0	-1
V_2	1	1	0	1/3	1/3	-2/3	0	1	-1
V_3	0	1	0	-1/3	2/3	-1/3	-1	1	0
V_4	0	1	1	-2/3	1/3	1/3	-1	0	1
V_5	0	0	1	-1/3	-1/3	2/3	0	-1	1
V_6	1	0	1	1/3	-2/3	1/3	1	-1	0
V_7	1	1	1	0	0	0	0	0	0

(Note that the respective voltage should be multiplied by V_{dc})

II. PWM METHODS
C. Space Vector PWM (5)

- **Principle of Space Vector PWM**

  - **Treats the sinusoidal voltage as a constant amplitude vector rotating at constant frequency**

  - **This PWM technique approximates the reference voltage V_{ref} by a combination of the eight switching patterns (V_0 to V_7)**

  - **CoordinateTransformation (abc reference frame to the stationary d-q frame)**
    : A three-phase voltage vector is transformed into a vector in the stationary d-q coordinate frame which represents the spatial vector sum of the three-phase voltage

  - **The vectors (V_1 to V_6) divide the plane into six sectors (each sector: 60 degrees)**

  - **V_{ref} is generated by two adjacent non-zero vectors and two zero vectors**

II. PWM METHODS

C. Space Vector PWM (6)

- **Basic switching vectors and Sectors**

  - **6 active vectors (V_1,V_2, V_3, V_4, V_5, V_6)**
    - **Axes of a hexagonal**
    - **DC link voltage is supplied to the load**
    - **Each sector (1 to 6): 60 degrees**

  - **2 zero vectors (V_0, V_7)**
    - **At origin**
    - **No voltage is supplied to the load**

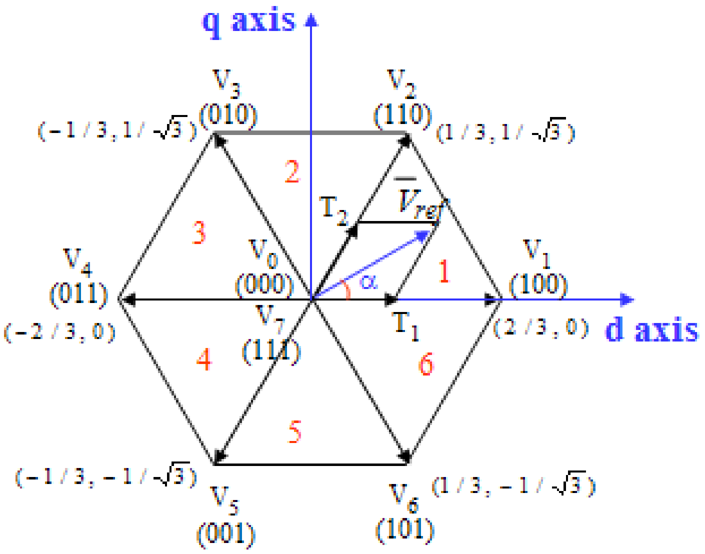


Fig. 11 Basic switching vectors and sectors.

II. PWM METHODS

C. Space Vector PWM (7)

➢ **Comparison of Sine PWM and Space Vector PWM (1)**

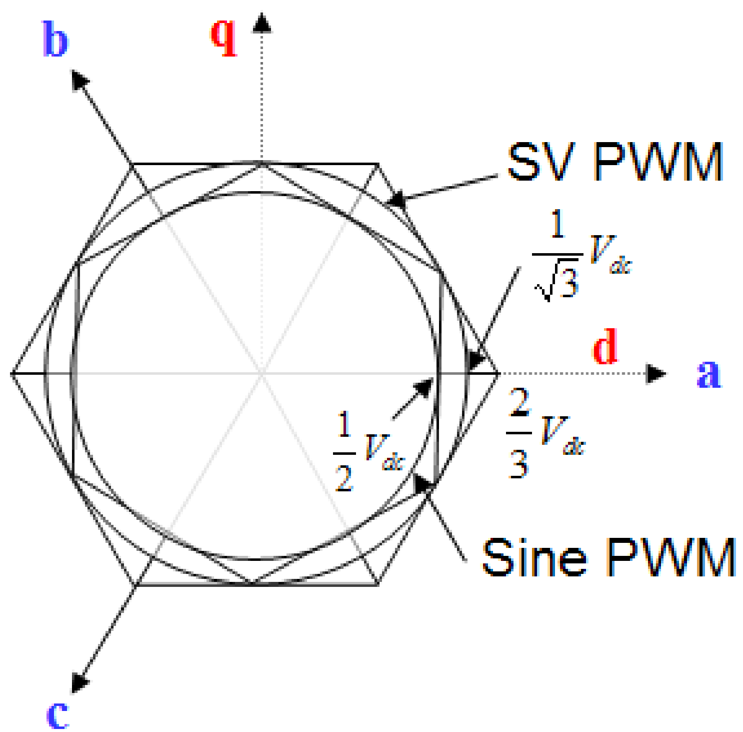


Fig. 12 Locus comparison of maximum linear control voltage in Sine PWM and SV PWM.

II. PWM METHODS
C. Space Vector PWM (8)

➢ **Comparison of Sine PWM and Space Vector PWM (2)**

- **Space Vector PWM generates less harmonic distortion in the output voltage or currents in comparison with sine PWM**

- **Space Vector PWM provides more efficient use of supply voltage in comparison with sine PWM**

  ⇨ **Sine PWM**

  : Locus of the reference vector is the inside of a circle with radius of 1/2 V_{dc}

  ⇨ **Space Vector PWM**

  : Locus of the reference vector is the inside of a circle with radius of $1/\sqrt{3}$ V_{dc}

∴ Voltage Utilization: Space Vector PWM = $2/\sqrt{3}$ times of Sine PWM

II. PWM METHODS

C. Space Vector PWM (9)

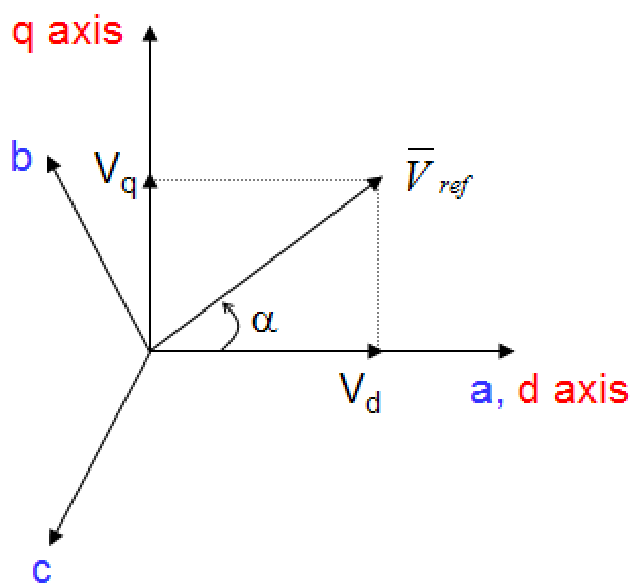
- **Realization of Space Vector PWM**

  - **Step 1.** **Determine V_d, V_q, V_{ref}, and angle (α)**

  - **Step 2.** **Determine time duration T_1, T_2, T_0**

  - **Step 3.** **Determine the switching time of each transistor (S_1 to S_6)**

- **Step 1. Determine V_d, V_q, V_{ref}, and angle (α)**
  - **Coordinate transformation**
    : abc to dq



$$V_d = V_{an} - V_{bn} \cdot \cos 60 - V_{cn} \cdot \cos 60$$
$$= V_{an} - \frac{1}{2} V_{bn} - \frac{1}{2} V_{cn}$$

$$V_q = 0 + V_{bn} \cdot \cos 30 - V_{cn} \cdot \cos 30$$
$$= V_{an} + \frac{\sqrt{3}}{2} V_{bn} - \frac{\sqrt{3}}{2} V_{cn}$$

$$\therefore \begin{bmatrix} V_d \\ V_q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}$$

$$\left| \overline{V}_{ref} \right| = \sqrt{V_d^{\,2} + V_q^{\,2}}$$

$$\alpha = \tan^{-1}\left(\frac{V_q}{V_d}\right) = \omega_s t = 2\pi f_s t$$

(where, f_s = fundamental frequency)

Fig. 13 Voltage Space Vector and its components in (d, q).

II. PWM METHODS

C. Space Vector PWM (11)

- **Step 2.** **Determine time duration T_1, T_2, T_0 (1)**

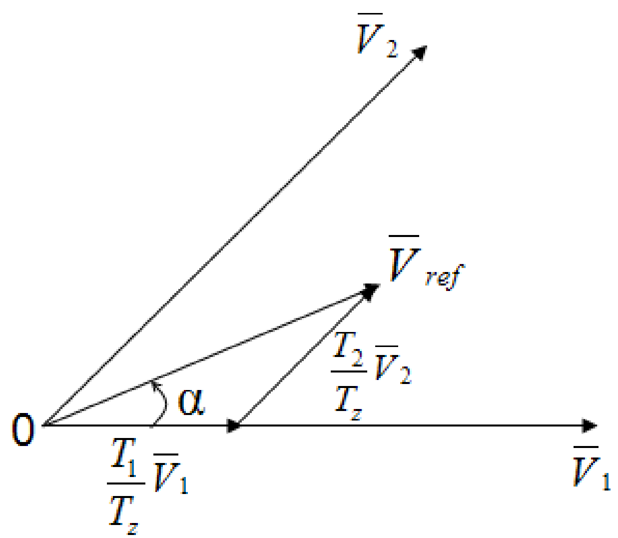


Fig. 14 Reference vector as a combination of adjacent vectors at sector 1.

II. PWM METHODS
C. Space Vector PWM (12)

➢ **Step 2. Determine time duration T_1, T_2, T_0 (2)**

- **Switching time duration at Sector 1**

$$\int_0^{T_z} \overline{V}_{ref} = \int_0^{T_1} \overline{V}_1 dt + \int_{T1}^{T_1+T_2} \overline{V}_2 dt + \int_{T_1+T_2}^{T_z} \overline{V}_0$$

$$\therefore T_z \cdot \overline{V}_{ref} = (T_1 \cdot \overline{V}_1 + T_2 \cdot \overline{V}_2)$$

$$\Rightarrow T_z \cdot \left|\overline{V}_{ref}\right| \cdot \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix} = T_1 \cdot \frac{2}{3} \cdot V_{dc} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + T_2 \cdot \frac{2}{3} \cdot V_{dc} \cdot \begin{bmatrix} \cos(\pi/3) \\ \sin(\pi/3) \end{bmatrix}$$

$$(\text{where}, \quad 0 \le \alpha \le 60°)$$

$$\therefore T_1 = T_z \cdot a \cdot \frac{\sin(\pi/3 - \alpha)}{\sin(\pi/3)}$$

$$\therefore T_2 = T_z \cdot a \cdot \frac{\sin(\alpha)}{\sin(\pi/3)}$$

$$\therefore T_0 = T_z - (T_1 + T_2), \quad \left(\text{where}, \quad T_z = \frac{1}{f_s} \quad \text{and} \quad a = \frac{\left|\overline{V}_{ref}\right|}{\frac{2}{3} V_{dc}} \right)$$

II. PWM METHODS

C. Space Vector PWM (13)

- **Step 2.** **Determine time duration T_1, T_2, T_0 (3)**

  - **Switching time duration at any Sector**

$$\therefore T_1 = \frac{\sqrt{3}\cdot T_z \cdot \left|\overline{V}ref\right|}{V_{dc}}\left(\sin\left(\frac{\pi}{3} - \alpha + \frac{n-1}{3}\pi\right)\right)$$

$$= \frac{\sqrt{3}\cdot T_z \cdot \left|\overline{V}ref\right|}{V_{dc}}\left(\sin\frac{n}{3}\pi - \alpha\right)$$

$$= \frac{\sqrt{3}\cdot T_z \cdot \left|\overline{V}ref\right|}{V_{dc}}\left(\sin\frac{n}{3}\pi\cos\alpha - \cos\frac{n}{3}\pi\sin\alpha\right)$$

$$\therefore T_2 = \frac{\sqrt{3}\cdot T_z \cdot \left|\overline{V}ref\right|}{V_{dc}}\left(\sin\left(\alpha - \frac{n-1}{3}\pi\right)\right)$$

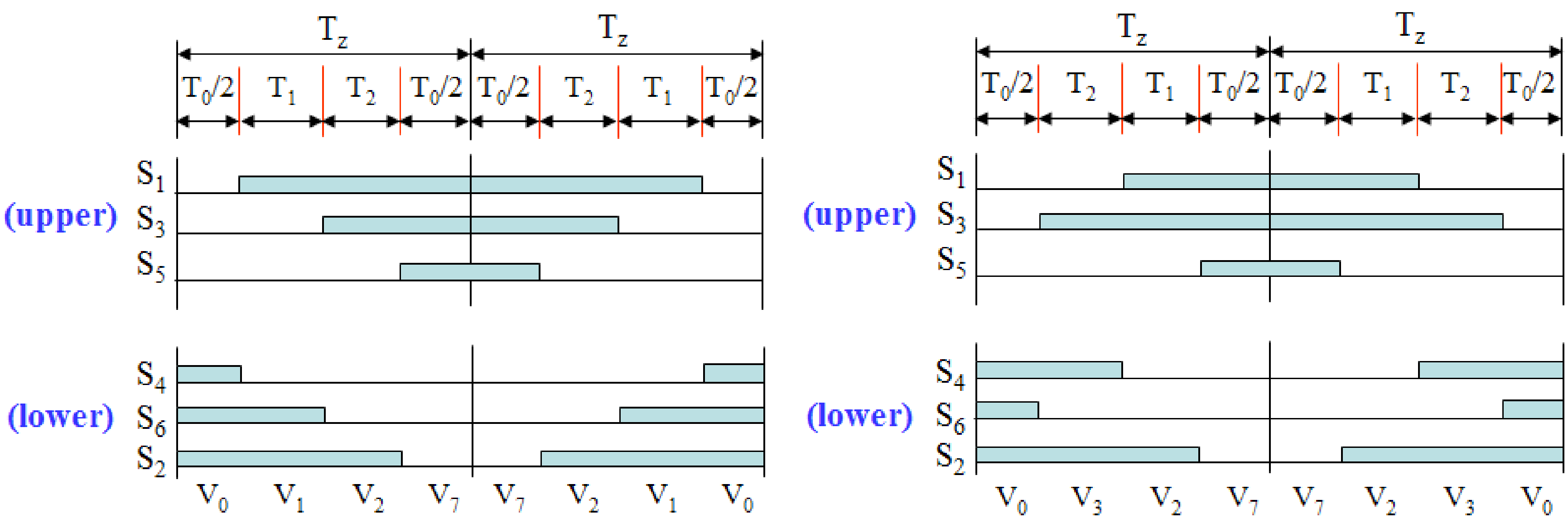
$$= \frac{\sqrt{3}\cdot T_z \left|\overline{V}ref\right|}{V_{dc}}\left(-\cos\alpha\cdot\sin\frac{n-1}{3}\pi + \sin\alpha\cdot\cos\frac{n-1}{3}\pi\right)$$

$$\therefore T_0 = T_z - T_1 - T_2, \quad \left(\begin{array}{c}\text{where, } \mathrm{n} = 1 \text{ through } 6 \text{(that is, Sector1 to 6)} \\ 0 \le \alpha \le 60^\circ\end{array}\right)$$

II. PWM METHODS

C. Space Vector PWM (14)

➢ **Step 3.** **Determine the switching time of each transistor (S_1 to S_6) (1)**

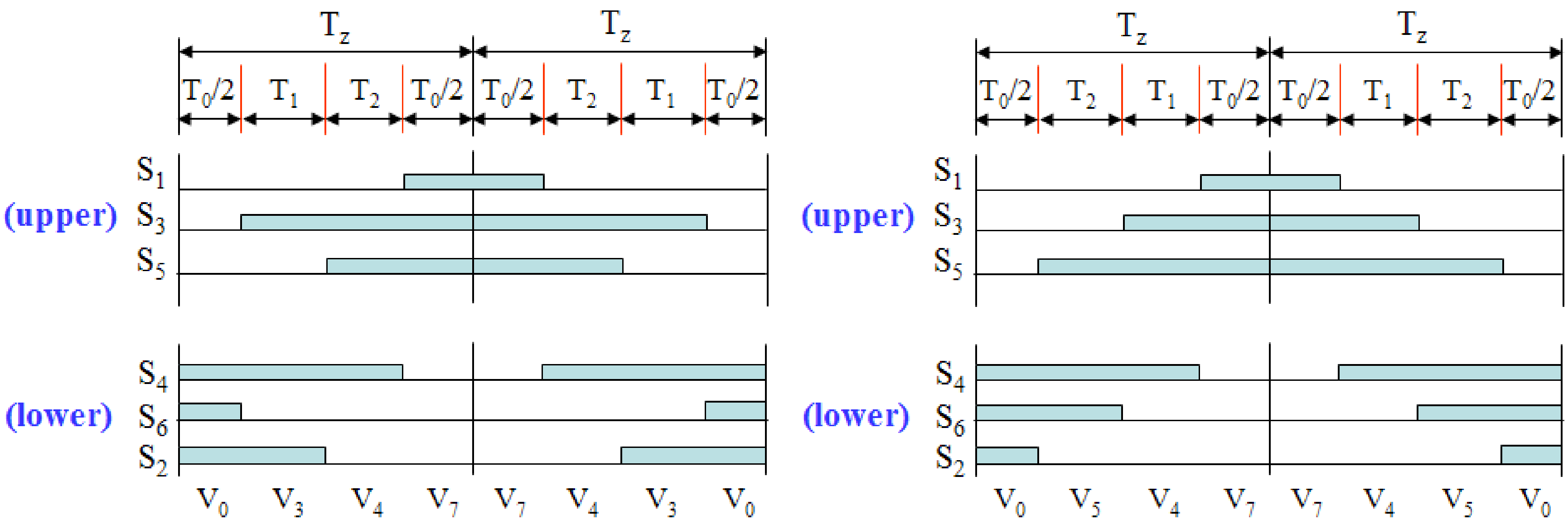


(a) Sector 1. **(b) Sector 2.**

Fig. 15 Space Vector PWM switching patterns at each sector.

II. PWM METHODS
C. Space Vector PWM (15)

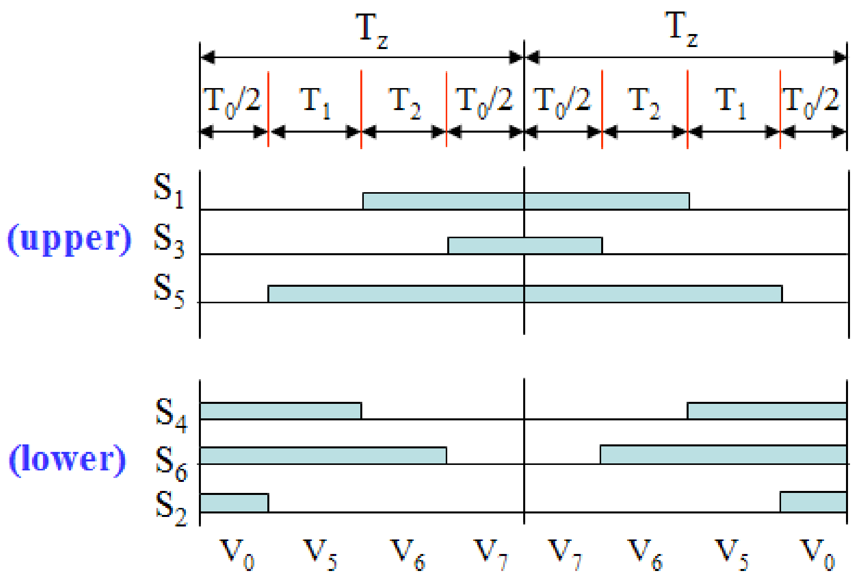
➢ **Step 3. Determine the switching time of each transistor (S_1 to S_6) (2)**



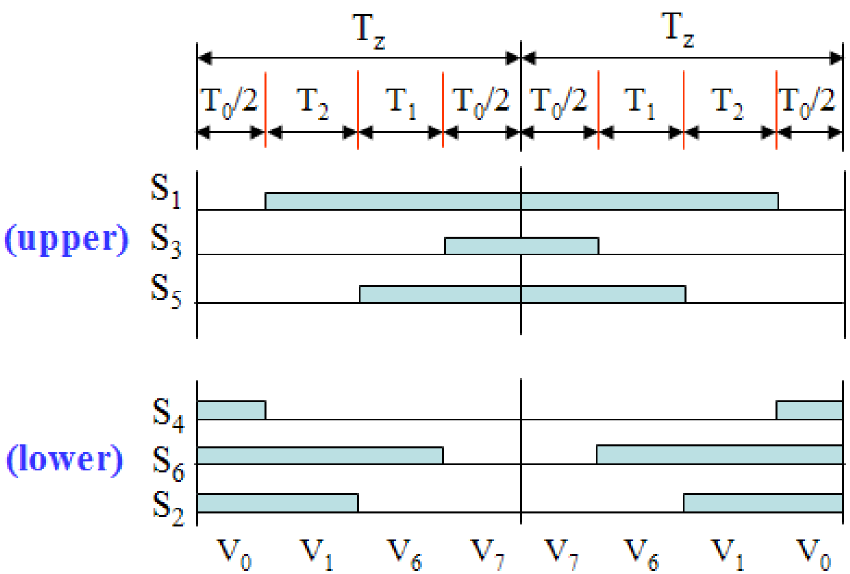
(c) Sector 3. (d) Sector 4.

Fig. 15 Space Vector PWM switching patterns at each sector.

➢ **Step 3.** **Determine the switching time of each transistor (S_1 to S_6) (3)**



(e) Sector 5.



(f) Sector 6.

Fig. 15 Space Vector PWM switching patterns at each sector.

II. PWM METHODS
C. Space Vector PWM (17)

➢ **Step 3.** **Determine the switching time of each transistor (S_1 to S_6) (4)**

Table 1. Switching Time Table at Each Sector

Sector	Upper Switches (S_1, S_3, S_5)	Lower Switches (S_4, S_6, S_2)
1	$S_1 = T_1 + T_2 + T_0/2$ $S_3 = T_2 + T_0/2$ $S_5 = T_0/2$	$S_4 = T_0/2$ $S_6 = T_1 + T_0/2$ $S_2 = T_1 + T_2 + T_0/2$
2	$S_1 = T_1 + T_0/2$ $S_3 = T_1 + T_2 + T_0/2$ $S_5 = T_0/2$	$S_4 = T_2 + T_0/2$ $S_6 = T_0/2$ $S_2 = T_1 + T_2 + T_0/2$
3	$S_1 = T_0/2$ $S_3 = T_1 + T_2 + T_0/2$ $S_5 = T_2 + T_0/2$	$S_4 = T_1 + T_2 + T_0/2$ $S_6 = T_0/2$ $S_2 = T_1 + T_0/2$
4	$S_1 = T_0/2$ $S_3 = T_1 + T_0/2$ $S_5 = T_1 + T_2 + T_0/2$	$S_4 = T_1 + T_2 + T_0/2$ $S_6 = T_2 + T_0/2$ $S_2 = T_0/2$
5	$S_1 = T_2 + T_0/2$ $S_3 = T_0/2$ $S_5 = T_1 + T_2 + T_0/2$	$S_4 = T_1 + T_0/2$ $S_6 = T_1 + T_2 + T_0/2$ $S_2 = T_0/2$
6	$S_1 = T_1 + T_2 + T_0/2$ $S_3 = T_0/2$ $S_5 = T_1 + T_0/2$	$S_4 = T_0/2$ $S_6 = T_1 + T_2 + T_0/2$ $S_2 = T_2 + T_0/2$

III. REFERENCES

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